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STEADY STATE POWER TRANSMISSION THROUGH A MULTILAYERED FERROELECTRIC DEVICE WITH ELECTROMECHANICAL DISSIPATION

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Abstract—This document presents solutions to the ferroelectric wave equation with boundary conditions for a one-dimensional multilayered device. The internal losses of the device are considered through the use of constitutive equations that include the time-dependent response of the materials. The constitutive equations were chosen with T and E as dependent variables so that output power could be calculated readily from the transducer geometry and the load. The results are in agreement with experimental data presented herein.

1. INTRODUCTION

The transmission of electric and acoustical energy through a multilayered ferroelectric device is described by the propagation of the electric and the acoustic waves from the first to the final layer of the device. As these waves propagate, it is possible for reflections to occur at all interfaces between layers so that only part of the incident energy is transmitted into the next layer. Reflections, however, account for only part of this loss of amplitude. Internal loss mechanisms, of which dielectric relaxation and viscous attenuation are well known examples, add several forms of damping that become very important at high frequencies and at all frequencies for which reflections are minimal. Like reflections, this material attenuation is frequency-dependent.

The major aspect of the design of such a device is the determination of the frequency for which the combined effects of reflections and material dissipation are minimum. The solution to the piezoelectric wave equation with appropriate boundary conditions can predict this frequency if the material dissipation is included. If this dissipation is not included, the solution is not bounded at frequencies of zero reflection and infinite output voltages and currents are predicted.

In this paper a one-dimensional model is analyzed. The wave equation and boundary conditions to be solved are appropriate for the five-layer one-dimensional device shown in Fig. 1. In this figure, layers 1 and 5 are the ferroelectric ceramic transducers; layers 2 and 4 are conductive bonds; and layer 3 is a conductive barrier. All materials are assumed to be

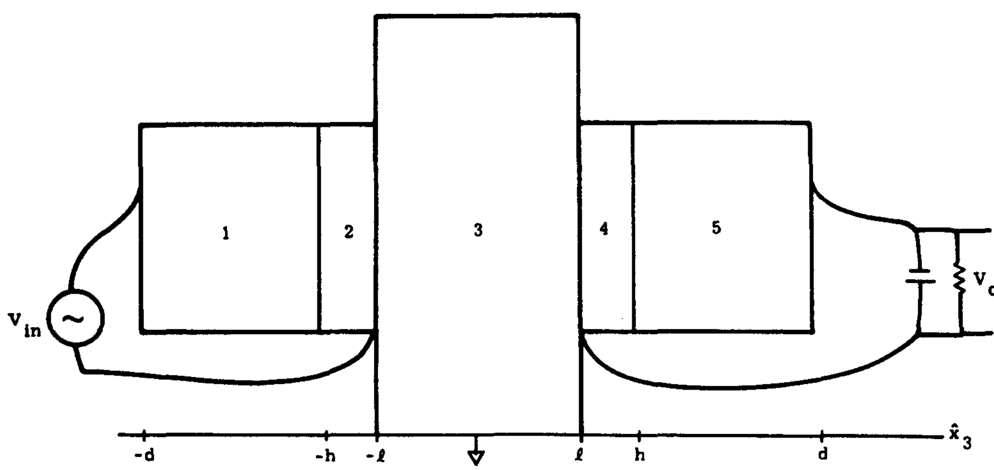

Fig. 1. Transducer geometry. Layers 1 and 5 are piezoelectric transducers. Layers 2 and 4 are conductive bonds. Layer 3 is a conductive barrier.

dissipative. The electroded surfaces at $x_3 = \pm d$, the conductivity of the bonds and the barrier and the smoothness of the bonds are considered ideal. In this analysis, the one-dimensional, longitudinal waves propagate in the x_3 direction. The boundary conditions account for the reflections and incomplete transmissions, while the attenuation caused by the materials themselves is included in the constitutive relations of the materials.

2. CONSTITUTIVE ASSUMPTIONS

The constitutive relations for the ceramic layers can be expressed as [1]

$$T(x,t) = c(O)S(x,t) + \int_0^t \frac{\mathrm{d}}{\mathrm{d}t} c(t-\tau)S(x,\tau)\,\mathrm{d}\tau - h_D(O)D(t) - \int_0^t \frac{\mathrm{d}}{\mathrm{d}t} h_D(t-\tau)D(\tau)\,\mathrm{d}\tau \tag{1}$$

and

$$E(x,t) = -h_S(O)S(x,t) - \int_0^t \frac{\mathrm{d}}{\mathrm{d}t} h_S(t-\tau)S(x,\tau)\,\mathrm{d}\tau + \beta(O)D(t) + \int_0^t \frac{\mathrm{d}}{\mathrm{d}t} \beta(t-\tau)D(\tau)\,\mathrm{d}\tau. \tag{2}$$

The functions c, h_D, h_S, and β are assumed to have exponential time-dependence. Also, single relaxation times are assumed [1]. Hence,

$$\begin{aligned} c(t) &= c_e - (c_e - c_i)\,\mathrm{e}^{-t/\tau_S}, \\ h_D(t) &= h_{De} - (h_{De} - h_{Di})\,\mathrm{e}^{-t/\tau_D}, \\ h_S(t) &= h_{Se} - (h_{Se} - h_{Si})\,\mathrm{e}^{-t/\tau_S}, \end{aligned} \tag{3}$$

and

$$\beta(t) = \beta_e - (\beta_e - \beta_i)\,\mathrm{e}^{-t/\tau_D},$$

where τ_S and τ_D are relaxation times for strain and electric displacements. The subscripts e and i refer to equilibrium and instantaneous values.

Substituting for c and h_D in eqn (1) and assuming

$$S(x,t) = S(x)\,\mathrm{e}^{i\omega t}$$

and

$$D(t) = D\,\mathrm{e}^{i\omega t}$$

(D is independent of x by Gauss's law for charge-free bodies),

$$T = c_i S(x)\,\mathrm{e}^{i\omega t} + \frac{c_e - c_i}{\tau_S} S(x)\,\mathrm{e}^{-t/\tau_S} \int_0^t \mathrm{e}^{(1/\tau_S + i\omega)\tau}\,\mathrm{d}\tau - h_{Di} D\,\mathrm{e}^{i\omega t} + \frac{h_{De} - h_{Di}}{\tau_D} D\,\mathrm{e}^{-t/\tau_D} \int_0^t \mathrm{e}^{(1/\tau_D + i\omega)\tau}\,\mathrm{d}\tau.$$

Similarly,

$$E = -h_{Si}S(x)\,e^{i\omega t} - \frac{h_{Se}-h_{Si}}{\tau_S}\,S(x)\,e^{-t/\tau_S}\int_0^t e^{(1/\tau_S+i\omega)\tau}\,d\tau$$

$$+\beta_i D\,e^{i\omega t} + \frac{\beta_e-\beta_i}{\tau_D}\,D\,e^{-t/\tau_D}\int_0^t e^{(1/\tau_D+i\omega)\tau}\,d\tau.$$

Performing the integrations and allowing $t\to\infty$ for the steady-state case considered here, eqns (1) and (2) can be written as

$$T = c^*S - h_D^*D \tag{4}$$

$$E = -h_S^*S + \beta^*D. \tag{5}$$

where

$$c^* = c_i + \frac{c_e-c_i}{1+\omega^2\tau_S^2} - \frac{i(c_e-c_i)\omega\tau_S}{1+\omega^2\tau_S^2}, \tag{6}$$

$$h_D^* = h_{Di} + \frac{h_{De}-h_{Di}}{1+\omega^2\tau_D^2} - \frac{i(h_{De}-h_{Di})\omega\tau_D}{1+\omega^2\tau_D^2}, \tag{7}$$

$$h_S^* = h_{Si} + \frac{h_{Se}-h_{Si}}{1+\omega^2\tau_S^2} - \frac{i(h_{Se}-h_{Si})\omega\tau_S}{1+\omega^2\tau_S^2}, \tag{8}$$

and

$$\beta^* = \beta_i + \frac{\beta_e-\beta_i}{1+\omega^2\tau_D^2} - \frac{i(\beta_e-\beta_i)\omega\tau_D}{1+\omega^2\tau_D^2}. \tag{9}$$

All of the values required in eqns (6)–(9) are not readily available. They are not specified by ceramic suppliers and in some cases have never been determined [2]. In addition, the measurement of these time dependent material parameters is not trivial. However, available data from suppliers and the literature can be used to obtain some reasonable values. Hopefully, experimentally determined material parameters for at least the commonly used ceramics will be available soon.

The elastic parameters c_e, c_i and τ_S have been measured for the ceramic PZT 65/35 [3]. For this case, c is a relaxation function with $c_e = 0.9748c_i$. The relaxation time is given as 0.14 μs at room temperature (22°C). It is assumed that the above relationship between c_e and c_i holds and that τ_S is the same for all ferroelectric materials to be considered here. Absolute values for c_i and c_e are determined by assuming that at 1 kHz, the c given by the manufacturer is equal to the real part of c^*, Re(c^*). Such an assumption can be justified by examining the frequency-dependence of the imaginary part, Im(c^*), given $\tau_S = 0.14$ μs. Values for c^* in terms of c given by the manufacturer are

$$\mathrm{Re}(c^*) = c_i + \frac{(0.9748-1.0)}{1+\omega^2\tau_S^2}\,c_i$$

$$= c_{given} \text{ for } \omega = 2\pi\times10^3 \text{ and } \tau_S = 0.14\ \mu\text{s}.$$

Solving for c_i,

$$c_i = 1.02585\ c_{given}.$$

Substituting for c_i in eqn (6) yields

$$c^* = \left[1.02585 + \frac{0.02585}{1+\omega^2\tau_S^2} + \frac{0.02585 i\omega\tau_S}{1+\omega^2\tau_S^2}\right]c_{given}. \tag{10}$$

Since no data is available for the piezoelectric parameters, the relative values for c_i and c_e above were used for the h's:

$$h_e - h_i = 0.0252 h_i$$
$$h_i = 1.02585 h_{\text{given}}$$
$$\tau_S = 0.14\ \mu\text{s}$$
$$\tau_D = 0.20\ \mu\text{s from a measurement of } \tau_D \text{ for PZT 65/35}$$

$$h_S{}^* = \left[1.02585 + \frac{0.02585}{1+\omega^2\tau_S{}^2} + \frac{0.02585 i\omega\tau_S}{1+\omega^2\tau_S{}^2}\right] h_{\text{given}} \tag{11}$$

$$h_D{}^* = \left[1.02585 + \frac{0.02585}{1+\omega^2\tau_S{}^2} + \frac{0.02585 i\omega\tau_S}{1+\omega^2\tau_S{}^2}\right] h_{\text{given}}. \tag{12}$$

Data for the dielectric properties are usually given in terms of ϵ. This poses no problem since $\beta = 1/\epsilon$. Manufacturers usually specify a loss tangent at some frequency, f_0.

$$\tan\delta = \epsilon_{\text{im}}/\epsilon_R$$

where ϵ_R and ϵ_{im} are the real and imaginary parts of the complex dielectric constant ϵ^* and where ϵ_i can be a negative number. From eqns (6)–(9),

$$\epsilon_R = \epsilon_i + \frac{\epsilon_e - \epsilon_i}{1+\omega^2\tau_D{}^2}$$

and

$$\epsilon_{\text{im}} = \frac{(\epsilon_e - \epsilon_i)\omega\tau_D}{1+\omega^2\tau_D{}^2}.$$

Given $\tan\delta$ and f_0, the ratio of $(\epsilon_e - \epsilon_i)$ to ϵ_i can be calculated. For example, for Channel 5500 material, $\tan\delta = 0.02$ and $\epsilon_c = 2.08523\epsilon_i$. Using

$$\epsilon_{\text{given}} = \text{Re}(\epsilon^*) \text{ at } f = 1.0\,\text{kHz and } \tau_D = 0.20\ \mu\text{s},$$
$$\epsilon_i = 0.47956\epsilon_{\text{given}}.$$

Solving for β:

$$\beta_e = 1/\epsilon_e$$
$$\beta_i = 1/\epsilon_i$$
$$\beta^* = \left[2.0852 - \frac{1.085}{1+\omega^2\tau_D{}^2} + \frac{1.085 i\omega\tau_D}{1+\omega^2\tau_D{}^2}\right]\epsilon_{\text{given}}^{-1}. \tag{13}$$

Equations (1) and (2) describe the conductive layers as well as the ceramic ones. In the conductive layers, however, the piezoelectric coupling coefficients h_D and h_S and the dielectric term β are zero. Material dissipation in these layers can also be handled similarly to that in the ceramic with the elastic coefficient c becoming the complex c^* where

$$c^* = c_i + \frac{c_e - c_i}{1+\omega^2\tau_S{}^2} - \frac{i(c_e - c_i)\omega\tau_S}{1+\omega^2\tau_S{}^2}.$$

Values for c_e, c_i, and τ_S must be substituted into this expression to complete the analysis. It is more common, however, to find the material dissipation expressed in terms of α dB/m instead of in terms of relaxation time τ_S and instantaneous and equilibrium values of the elastic coefficient. In these cases, the material dissipation can be handled by defining the complex elastic constant in terms of α and the propagation constant k_l instead of in terms of τ_S, c_e and c_i. This procedure is described in the next section.

3. THE BOUNDARY VALUE PROBLEM

From eqn (4), the wave equations for all five layers can be written as

$$\frac{\partial^2 u_j}{\partial x^2} + \Omega_j^2 \omega^2 u_j = 0, \quad j = 1, 2, 3, 4, 5 \tag{14}$$

where

$$\frac{\partial T}{\partial x} = \rho \frac{\partial^2 u}{\partial t^2} \text{ (Newton's second law)}$$

and

$$\frac{\partial D}{\partial x} = 0 \text{ (Gauss's law for charge-free regions);}$$

and where

$$u_j = u_j(x)\, e^{i\omega t} \text{ (particle displacement)}$$

$$S_j = \frac{\partial u_j}{\partial x}$$

$$\Omega_j^2 = \rho_j / c_j^*, \ \rho_j = \text{density in the } j\text{th layer.}$$

For ease of the solution, all bonding materials and all ferroelectric materials are the same:

$$\Omega_1 = \Omega_5 \text{ and } \Omega_2 = \Omega_4.$$

The conductive layers are also taken to be isotropic. Material dissipation in these layers can be considered by redefining $\Omega\omega$ in the following manner[4].

$$\Omega^2\omega^2 = \omega^2 \rho_j / c_j^* = \text{the complex propagation constant squared}$$
$$= (k_l - i\alpha)^2,$$

where

$$k_l = \text{propagation constant for a lossy medium}$$
$$\cong \omega\sqrt{\rho_j/c_j}\left(1.0 - \frac{3}{8}\left(\frac{\omega\xi_j}{c_j}\right)^2\right)^{-1/2},$$

$$\alpha = \text{attenuation constant in nepers/m}$$
$$\cong \frac{\omega^2}{2}\sqrt{\rho_j/c_j}(\xi_j/c_j)$$

and $c_j = (\lambda_j + 2\mu_j)$ for longitudinal propagation in an isotropic medium. Solving for c_j^*,

$$c_j^* = \omega^2 \frac{\rho_j}{(k_l - i\alpha)^2}, \quad j = 2, 3, 4.$$

Solutions are assumed to be of the form

$$
\begin{aligned}
u_1(x) &= a_1 \cos \Omega_1\omega(x+h) + b_1 \sin \Omega_1\omega(x+h) \\
u_2(x) &= a_2 \cos \Omega_2\omega(x+l) + b_2 \sin \Omega_2\omega(x+l) \\
u_3(x) &= a_3 \cos \Omega_3\omega x + b_3 \sin \Omega_3\omega x \\
u_4(x) &= a_4 \cos \Omega_2\omega(x-l) + b_4 \sin \Omega_2\omega(x-l) \\
u_5(x) &= a_5 \cos \Omega_1\omega(x-h) + b_5 \sin \Omega_1\omega(x-h),
\end{aligned} \tag{15}
$$

where u_n = particle displacement in the nth layer.

Boundary conditions are written for stress-free surfaces at $x = \pm d$ and for continuity of stress and displacement at $x = \pm h$ and $x = \pm l$.

$$
\begin{aligned}
T_1(-d) &= 0 \\
T_5(d) &= 0 \\
T_1(-h) &= T_2(-h) \\
u_1(-h) &= u_2(-h) \\
T_2(-l) &= T_3(-l) \\
u_2(-l) &= u_3(-l) \\
T_3(l) &= T_4(l) \\
u_3(l) &= u_4(l) \\
T_4(h) &= T_5(h) \\
u_4(h) &= u_5(h).
\end{aligned} \tag{16}
$$

Substituting for T, S, and u, the boundary conditions are:

$$
\begin{aligned}
& c_1^*\{a_1\Omega_1\omega \sin \Omega_1\omega\Delta + b_1\Omega_1\omega \cos \Omega_1\omega\Delta\} = h_D^*D_1 \\
& c_1^*\{b_1\Omega_1\omega\} - c_2^*\{a_2\Omega_2\omega \sin \Omega_2\omega\delta + b_2\Omega_2\omega \cos \Omega_2\omega\delta\} = h_D^*D_1 \\
& a_1 - a_2 \cos \Omega_2\omega\delta + b_2 \sin \Omega_2\omega\delta = 0 \\
& c_2^*\{b_2\Omega_2\omega\} - c_3^*\{a_3\Omega_3\omega \sin \Omega_3\omega l + b_3\Omega_3\omega \cos \Omega_3\omega l\} = 0 \\
& a_2 - a_3 \cos \Omega_3\omega l + b_3 \sin \Omega_3\omega l = 0 \\
& c_2^*\{b_2\Omega_2\omega\} + c_3^*\{a_3\Omega_3\omega \sin \Omega_3\omega l - b_3\Omega_3\omega \cos \Omega_3\omega l\} = 0 \\
& a_4 - a_3 \cos \Omega_3\omega l - b_3 \sin \Omega_3\omega l = 0 \\
& c_1^*\{b_5\Omega_1\omega\} + c_2^*\{a_4\Omega_2\omega \sin \Omega_2\omega\delta - b_4\Omega_4\omega \cos \Omega_4\omega\delta\} = h_D^*D_5 \\
& a_5 - a_4 \cos \Omega_2\omega\delta - b_4 \sin \Omega_2\omega\delta = 0 \\
& c_1^*\{-a_5\Omega_1\omega \sin \Omega_1\omega\Delta + b_5\Omega_1\omega \cos \Omega_1 u\Delta\} = h_D^*D_5.
\end{aligned} \tag{17}
$$

where

$$
\begin{aligned}
\Delta &= d - h, \\
\delta &= h - l,
\end{aligned}
$$

and D_n = electric displacement in the nth layer.

The electric displacement can be expressed in terms of input or output voltage by using

$$V = -\int_{x_1}^{x_2} E \, dx.$$

Substituting for E from eqn (5) and using $D \neq D(x)$,

$$V_{in} = h_S{}^* \int_{-d}^{-h} \left(\frac{du_1}{dx}\right) dx - \beta^* \Delta D.$$

Substituting for u_1 from eqn (15) and rearranging,

$$D_1 = \frac{-V_{in} + h_S{}^*\{a_1(1-\cos \Omega_1\omega\Delta) + b_1 \sin \Omega_1\omega\Delta\}}{\beta^*\Delta}.$$

Similarly,

$$D_5 = \frac{-V_{out} + h_S{}^*\{-a_5(1-\cos \Omega_1\omega\Delta) + b_5 \sin \Omega_1\omega\Delta\}}{\beta^*\Delta}.$$

The voltage input is specified by the problem. The output voltage depends on the load. For the device in Fig. 1.

$$V_{out} = ZI_{out},$$

where Z = impedance of the load assuming $e^{i\omega t}$ time dependence $= ((1/R) + j\omega c)^{-1}$,

$$I_{out} = A_5 \frac{dD_5}{dt} = i\omega A_5 D_5,$$

and A_5 = area of the electrode on layer 5. Substituting and rearranging,

$$D_5 = \frac{h_D{}^*\{-a_5(1-\cos \Omega_1\omega\Delta) + b_5 \sin \Omega_1\omega\Delta\}}{\beta^*\Delta + i\omega Z A_5}$$

The final form for the boundary conditions is obtained by substituting for D from eqns (18) and (19) and nondimensionalizing:

$$a_1\left\{\eta_1 \sin \eta_1 - \frac{h_D{}^* h_S{}^*}{c_1{}^*\beta^*}(1-\cos \eta_1)\right\} + b_1\left\{\eta_1 \cos \eta_1 - \frac{h_D{}^* h_S{}^*}{c_1{}^*\beta^*}\sin \eta_1\right\} = \frac{-h_D{}^* V_{in}}{\beta^* c_1{}^*},$$

$$a_1\left\{\frac{-h_D{}^* h_S{}^*}{c_1{}^*\beta^*}(1-\cos \eta_1)\right\} + b_1\left\{\eta_1 - \frac{h_D{}^* h_S{}^*}{c_1{}^*\beta^*}\sin \eta_1\right\} - a_2\left\{\frac{c_2{}^*\Delta}{c_1{}^*\delta}\eta_2 \sin \eta_2\right\} - b_2\left\{\frac{c_2{}^*\Delta}{c_1{}^*\delta}\eta_2 \cos \eta_2\right\} = \frac{-h_D{}^* V_{in}}{\beta^* c_1{}^*}$$

$$a_1 - a_2 \cos \eta_2 + b_2 \sin \eta_2 = 0,$$

$$b_2\left\{\frac{c_2{}^* l}{c_3{}^*\delta}\eta_2\right\} - a_3\{\eta_3 \sin \eta_3\} - b_3\{\eta_3 \cos \eta_3\} = 0$$

$$a_2 - a_3 \cos \eta_3 + b_3 \sin \eta_3 = 0$$

$$b_4\left\{\frac{c_2{}^* l}{c_3{}^*\delta}\eta_2\right\} + a_3\{\eta_3 \sin \eta_3\} - b_3\{\eta_3 \cos \eta_3\} = 0$$

$$a_4 - a_3 \cos \eta_3 - b_3 \sin \eta_3 = 0$$

$$a_5\left\{\frac{h_D{}^* h_S{}^*\delta}{c_2{}^*(\beta^*\Delta + i\omega Z A)}(1-\cos \eta_1)\right\} + b_5\left\{\frac{c_1{}^*\delta}{c_2{}^*\Delta}\eta_1 - \frac{h_D{}^* h_S{}^*\delta}{c_2{}^*(\beta^*\Delta + i\omega Z A)}\sin \eta_1\right\}$$

$$+ a_4\{\eta_2 \sin \eta_2\} - b_4\{\eta_2 \cos \eta_2\} = 0$$

$$a_5 - a_4 \cos \eta_2 - b_4 \sin \eta_2 = 0$$

$$a_5\left\{- \eta_1 \sin \eta_1 + \frac{h_S{}^*h_D{}^*\Delta}{c_1{}^*(\beta^*\Delta + i\omega ZA)}(1 - \cos \eta_1)\right\}$$

$$+ b_5\left\{\eta_1 \cos \eta_1 \frac{h_S{}^*h_D{}^*\Delta}{c_1{}^*(\beta^*\Delta + i\omega ZA)} \sin \eta_1\right\} = 0,$$

where

$$\eta_1 = \Omega_1\omega\Delta$$

$$\eta_2 = \Omega_2\omega\delta$$

$$\eta_3 = \Omega_3\omega l$$

$$\Delta = d - h$$

$$\delta = h - l.$$

The solution to this problem (i.e. the determination of the output voltage and current) is obtained by a computer program. For design purposes, the program is arranged to locate resonances by searching through a frequency band for power peaks.

4. EXAMPLE

A device similar to the one shown in Fig. 1 was built and tested. The device used Channel 5500 ceramic discs, conductive epoxy bonds and a 51-mil aluminum barrier. The ceramic discs were 0.5 in. thick and 1.5 in. in diameter. The bonds were approximately 6 mils thick. The output load for this device consisted of the 1.0-MΩ 20-pF input impedance of the storage oscilloscope used in obtaining the data. The resonant frequencies for this device and the amplitudes of the output voltages at these frequencies are presented in Fig. 2. The resonant

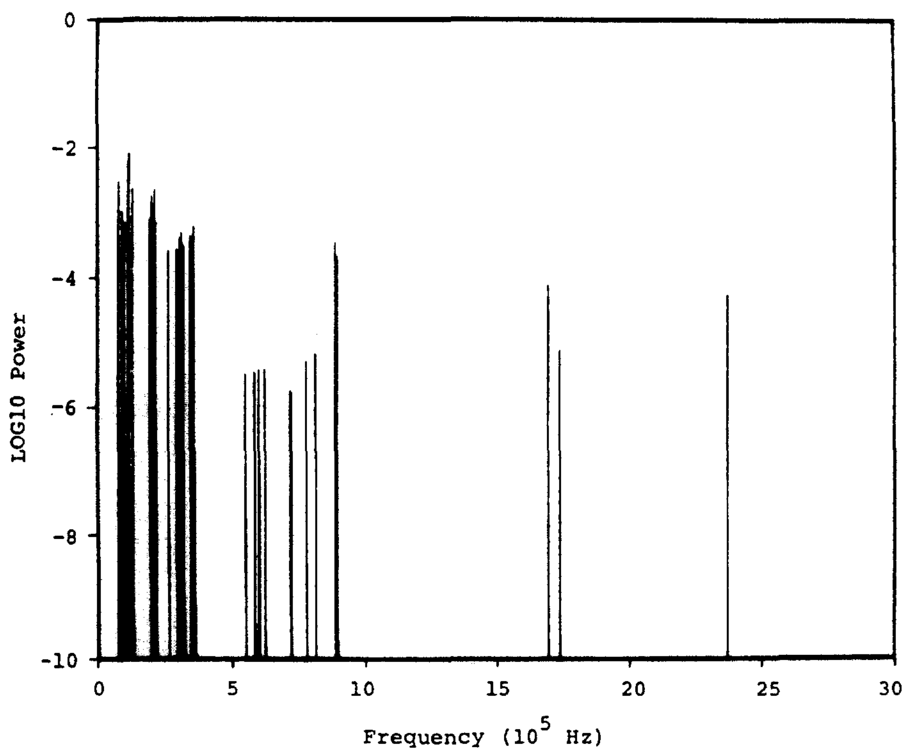

Fig. 2. Measured resonances. Data show slightly broader resonances and small amounts of power transmitted between resonances. Maximum power is transmitted at 117.0 kHz.

frequencies were located by sweeping the applied signal through a frequency range and observing the variations in output voltage and current from the device. The amplitudes of the resonant voltages were measured with the input signal fixed to the corresponding resonant frequency.

The numerical solution of the one-dimensional wave equation described in this paper predicted similar resonant frequencies and amplitudes (Fig. 3). This solution used the following parameters in its model of the experimental device [5, 8]

$$c_1 = 1.4497 \times 10^{11}$$
$$c_2 = 1.0287 \times 10^{10}$$
$$c_3 = 11.83 \times 10^{10}$$
$$h = 2.1499 \times 10^9 \text{ (ceramic only)}$$
$$\epsilon = 7.349 \times 10^{-9} \text{ (ceramic only)}$$
$$\rho_1 = 7600.0 \text{ kg/m}^3$$
$$\rho_2 = 1620.0 \text{ kg/m}^3$$
$$\rho_3 = 2695.0 \text{ kg/m}^3$$
$$\alpha_2 = 0.0582 \text{ nepers/mm at 1 MHz}$$
$$\alpha_3 = 7500 \text{ dB/m at 1 GHz}$$
$$\Delta = 6.35 \times 10^{-3} \text{ m}$$
$$\delta = 1.4 \times 10^{-4} \text{ m}$$
$$l = 0.6477 \times 10^{-3} \text{ m}$$

The results of two other calculations are shown in Figs. 4 and 5. In one of these calculations

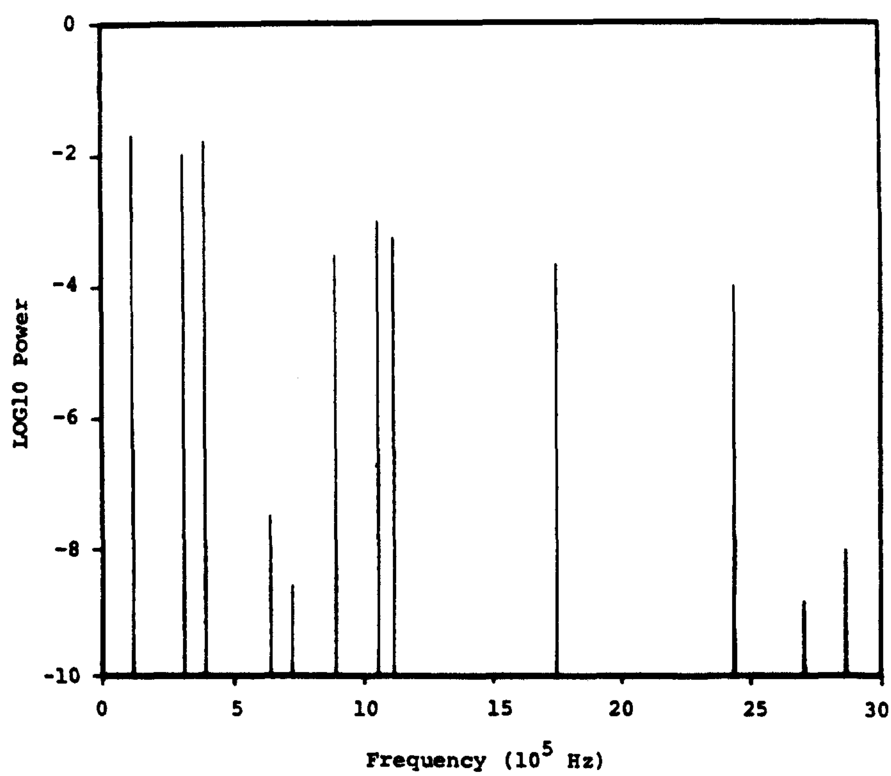

Fig. 3. Calculated resonances. Effects of mechanical dissipation and bonds are included. Maximum power is transmitted at 114.7 kHz.

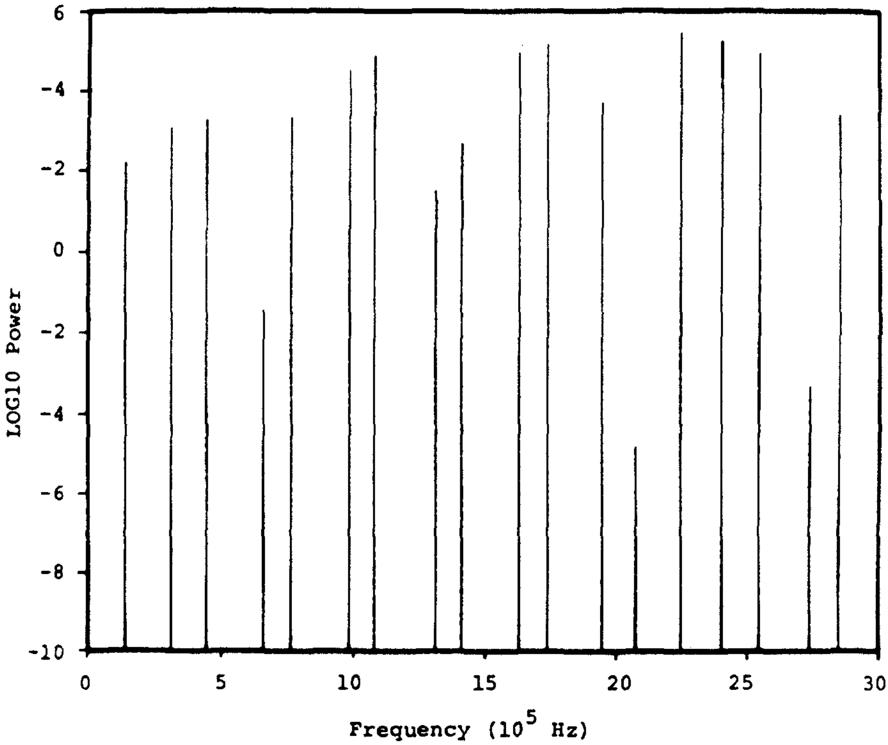

Fig. 4. Calculated resonances without the effects of mechanical dissipation and bonds. Values shown are the last finite value calculated before resonance. Note that some values are in the kilowatts range. Maximum power is transmitted at 2.2 MHz.

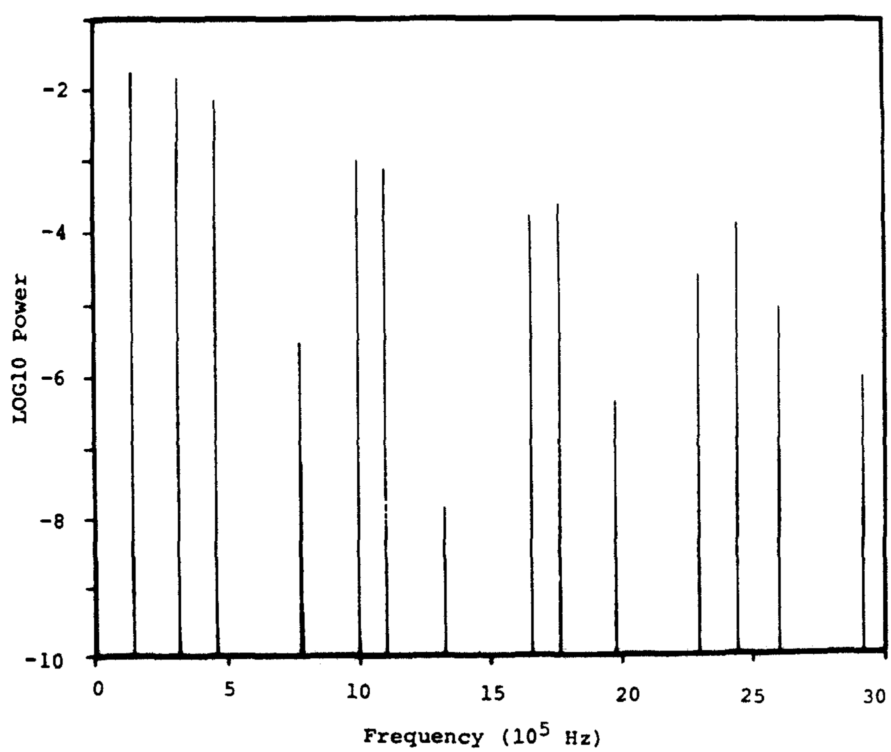

Fig. 5. Calculated resonances with the effects of mechanical dissipation. The effect of bonds is ignored. Maximum power is transmitted at 146.0 kHz.

(Fig. 4) the effects of the bonds and the material dissipation were not included [8]. Note that for this case, the amplitudes of all resonances are infinite. The finite values shown are the last finite values calculated before resonance. The results shown in Fig. 5 were obtained by solving the wave equation described in this paper with $\delta = 0$ so that the effect of the bonds was not included.

Figures 2–5 show only the resonant peaks. To display the data, power transmission at frequencies other than the resonant frequencies is set to ~0.0. In both the measured and the calculated cases, resonances are broader and between resonances some power is transmitted.

5. CONCLUSION

Figures 2 and 3 illustrate that the theoretical method presented in this paper has a high positive correlation with experimental data. The numerical solution could be improved by obtaining proper values for relaxation time and for other material parameters, especially in the piezoelectric case. Nonlinear attenuation is also an important factor to be considered because of its potential effect on energy transmission at resonance.

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